

Homework #6  
Chem 121

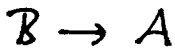
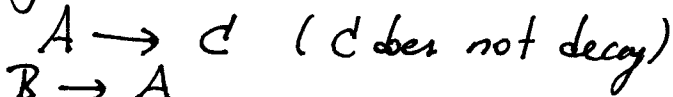
①

1. In the differential equation  $\frac{dy}{dx} = g(y/x)$   
show that the substitution  $u = y/x$  leads to a separable  
differential equation in  $u$  and  $x$ .

2. Radioactive nuclei decay according to the law

$$\frac{dN}{dt} = -\lambda N,$$

where  $N$  is the number of a given type of nucleus and  $\lambda$  is called the decay constant. If there are two types of radioactive nuclei



Then we can write:  $\frac{dN_A}{dt} = -\lambda_A N_A$

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B$$

Find  $N_B(t)$  with initial conditions  $N_A(0) = N_0$  and

$$N_B(0) = 0$$

3. Laguerre's Differential equation is

$$x L_n''(x) + (1-x) L_n'(x) + n L_n(x) = 0$$

Find a series solution. For what values of  $n$  will your answer be a polynomial with a finite highest power?

NB: Recall that this problem appears in the QM of the H-atom.

\* 4. The decay of a population by catastrophic two body collisions is described by

$$\frac{dN}{dt} = -kN^2$$

Solve this differential equation with initial condition  $N(0) = N_0$ .

5. (a) Develop a series solution for Hermite's equation

$$y'' - 2xy' + 2\alpha y = 0$$

where  $\alpha$  is a constant.

(b) Show that both series solutions are convergent by checking that the ratio of successive terms go as  $\text{like } e^{-2x^2}$  for large index.

(c) Find an appropriate choice of  $\alpha$  so that the series terminate

These polynomials are found in the solutions of the wavefunctions for a quantum harmonic oscillator and are called Hermite polynomials.