Homework #6
Chem 121

1. In the differential equation $\frac{dy}{dx} = g(y/x)$

Show that the substitution $u = y/x$ leads to a <u>separable</u> differential equation in u and x.

2. Radioactive nuclei decay according to the law

$$\frac{dN}{dt} = -\lambda N,$$

where N is the number of a given type of nucleus and λ is called the decay constant. If there are two types of radioactive nuclei

$A \longrightarrow C$ (C does not decay)
$B \rightarrow A$

Then we can write:

$$\frac{dN_A}{dt} = -\lambda_A N_A$$

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B$$

Find $N_B(t)$ with initial conditions $N_A(0) = N_0$ and

$$N_B(0) = 0$$

3. Laguerre's Differential equation is

$$x L_n''(x) + (1-x) L_n'(x) + n L_n(x) = 0$$

Find a series solution. For what values of n will your answer be a polynomial with a finite highest power?

NB: Recall that this problem appears in the QM of the H-atom.

* 4. The decay of a population by catastrophic two body collisions is described by

$$\frac{dN}{dt} = -kN^2$$

Solve this differential equation with initial condition $N(0) = N_0$.

5.(a) Develop a series solution for Hermite's equation

$$y'' - 2xy' + 2\alpha y = 0$$

where α is a constant.

(b) Show that <u>both</u> series solutions are convergent by checking that the ratio of successive terms go as those of e^{2x^2} for large index.

(c) Find an appropriate choice of α so that the series ~~terminate~~ These polynomials are found in the solution of the wavefunctions for a quantum harmonic oscillator and are called Hermite polynomials.